DEPENDENCES FOR THE EQUILIBRIUM FLOW OF DISSOCIATING GAS IN THE TURBINE CHANNELS OF NUCLEAR POWER PLANTS

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UDC 533.6.011:539.196.6

By representing the desired dependences for an arbitrary dissociating gas in terms of the parameters p and T, expressions are obtained in analytic form for the heat drop in the turbine and its stages.

The equation for equilibrium flow of dissociating gas, taking nonideality, energy transfer, and friction into account, will be considered.

1. To derive general dependences with an arbitrary equation of state, the generalized Bernoulli equation is used [1], in the form

$$\frac{dp}{\rho}+d\frac{w^2}{2}+dL_{\text{tech}}+dL_{\text{fr}}=0 \tag{1}$$

or

$$\int_1^2\frac{dp}{\rho}+\frac{w_2^2-w_1^2}{2}+L_{\text{tech}}+L_{\text{fr}}=0. \tag{2}$$

The equation of state for a dissociating gas is written in general form

$$pv=Z_{\text{ef}}\frac{R}{\mu}T, \tag{3}$$

where Z_{ef} takes account both of the dissociation factor and of the nonideality of the gas. Equation (3) shows that the product pv is a function of p and T

$$pv=f(p,\ T). \tag{4}$$

Hence

$$\left(\frac{\partial f}{\partial p}\right)_T dp=\left[\frac{\partial (pv)}{\partial p}\right]_T dp=p\left(\frac{\partial v}{\partial p}\right)_T dp+vdp.$$

Substituting vdp = dp/ρ into Eq. (1) then gives

$$d\frac{w^2}{2}+\left(\frac{\partial f}{\partial p}\right)_T dp-p\left(\frac{\partial v}{\partial p}\right)_T dp+dL_{\text{tech}}+dL_{\text{fr}}=0. \tag{5}$$

The result of differentiating Eq. (3) is

$$\left(\frac{\partial v}{\partial p}\right)_T=-\frac{RT}{\mu p^2}\left[Z_{\text{ef}}-p\left(\frac{\partial Z_{\text{ef}}}{\partial p}\right)_T\right]. \tag{6}$$

Introducing the notation

$$\eta=Z_{\text{ef}}-p\left(\frac{\partial Z_{\text{ef}}}{\partial p}\right)_T, \tag{7}$$

Eq. (5) is written in the form

$$d\frac{w^2}{2}+\left(\frac{\partial f}{\partial p}\right)_T dp+\frac{RT}{\mu}\eta\frac{dp}{p}+dL_{\text{tech}}+dL_{\text{fr}}=0. \tag{8}$$

The thermodynamic differential equation is chosen in the form

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M. Azizbekov Azerbaidzhan Institute of Petroleum and Chemistry, Baku. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 42, No. 6, pp. 982-987, June, 1982. Original article submitted March 30, 1981.

0022-0841/82/4206-0665$07.50

$$TdS = C_p dT - T\left(\frac{\partial v}{dT}\right)_p dp. \tag{9}$$

On the basis of Eq. (3), the following relation may be written

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{\mu p}\left[Z_{\text{ef}} + T\left(\frac{\partial Z_{\text{ef}}}{\partial T}\right)_p\right] \tag{10}$$

or after introducing the notation

$$\omega = Z_{\text{ef}} + T\left(\frac{\partial Z_{\text{ef}}}{\partial T}\right)_p, \tag{11}$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{\mu p}\,\omega. \tag{12}$$

The thermodynamic differential equation is known in the form

$$TdS = C_p dT - \left(\frac{\partial v}{\partial T}\right)_p dp. \tag{13}$$

Simultaneous solution of Eqs. (9) and (12) gives the expression

$$\frac{dp}{p} = \frac{1}{\omega}\,\frac{C_p dT - TdS}{TR/\mu},$$

which leads, when substituted into Eq. (8), to the result

$$d\,\frac{w^2}{2} + \left(\frac{\partial f}{\partial p}\right)_T dp + \frac{\eta}{\omega}\left(C_p dT - TdS\right) + dL_{\text{tech}} + dL_{\text{fr}} = 0. \tag{14}$$

The quantity ω is related to the isobaric specific heat and the temperature index of the adiabatic. As is known [2, 3]

$$\left(\frac{\partial p}{\partial v}\right)_s = -k_{\text{so}}\,\frac{p}{v}, \tag{15}$$

$$\left(\frac{\partial T}{\partial p}\right)_s = \frac{k_T - 1}{k_T}\,\frac{T}{p}. \tag{16}$$

But

$$\left(\frac{\partial T}{\partial p}\right)_s = \frac{T}{C_p}\left(\frac{\partial v}{\partial T}\right)_p. \tag{17}$$

Then, taking Eq. (10) into account, it is found that

$$\omega = \frac{C_p}{R/\mu}\,\frac{k_T - 1}{k_T}. \tag{18}$$

In the light of the thermodynamic equation

$$TdS = dh - \frac{dp}{\rho} \tag{19}$$

and of Eq. (18), Eq. (14) may be rewritten in the form

$$d\,\frac{w^2}{2} + \left(\frac{\partial f}{\partial p}\right)_T dp + \frac{R}{\mu}\,\frac{k_T}{k_T - 1}\,\eta\left[dT - \frac{1}{C_p}\left(dh - \frac{dp}{\rho}\right)\right] + dL_{\text{tech}} + dL_{\text{fr}} = 0. \tag{20}$$

As is known, the total amount of heat $dQ = dQ_{\text{sup}} + dQ_{\text{fr}}$, while $dQ_{\text{fr}} = dL_{\text{fr}}$.

The expression for the first law of thermodynamics applicable to gas moving with respect to the coordinates is written in the form

$$dQ = dh + d\,\frac{w^2}{2} + dL_{\text{tech}} + dL_{\text{fr}}$$

or when Eq. (1) is taken into account

$$dh - \frac{dp}{\rho} = dQ. \tag{21}$$

Substituting Eq. (21) into Eq. (20) gives

$$d\frac{w^2}{2}+\left(\frac{\partial f}{\partial p}\right)_T dp+\frac{R}{\mu}\frac{k_T}{k_T-1}\eta\left(dT-\frac{dQ}{C_p}\right)+dL_{\text{tech}}+dL_{\text{fr}}=0. \tag{22}$$

The quantities η and $k_T/(k_T - 1)$ change insignificantly in the course of gas expansion, and therefore their mean values may be used in the integration. For convenience of notation, the averaging bar above $k_T/(k_T - 1)$ is omitted. Integrating Eq. (22) gives

$$\frac{w_2^2-w_1^2}{2}+\int\limits_1^2\left(\frac{\partial f}{\partial p}\right)_T+\frac{R}{\mu}\frac{k_T}{k_T-1}\bar{\eta}(T_2-T_1)-\frac{\bar{\eta}}{\bar{\omega}}Q_{\text{sup}}+L_{\text{tech}}+L_{\text{fr}}=0. \tag{23}$$

In the absence of heat supply in all cases (Q = 0), forms of Eq. (23) appropriate for the nozzle network, working blades, and turbine as a whole, with the corresponding indices: for the nozzle network ($L_{\text{tech}} = 0$)

$$\frac{w_1^2-w_0^2}{2}+\int\limits_0^1\left(\frac{\partial f}{\partial p}\right)_T dp+\frac{R}{\mu}\frac{k_T}{k_T-1}\bar{\eta}(T_1-T_0)+L_{\text{fr}}^{\text{NN}}=0, \tag{24}$$

for the working blades of the stages

$$\frac{w_2^2-w_1^2}{2}+\int\limits_1^2\left(\frac{\partial f}{\partial p}\right)_T dp+\frac{R}{\mu}\frac{k_T}{k_T-1}\bar{\eta}(T_2-T_1)+L_{\text{fr}}^{\text{WB}}+L_{\text{WB}}=0. \tag{25}$$

For turbines, w_1 and w_2 in Eq. (23) may be understood to mean, respectively, the velocity of gas supplied to the turbine and at the turbine outlet. Taking $w_2 \approx w_1$ [4], and denoting the gas parameters before reaching the turbine by the subscript 0 and those at the turbine outlet by 2, the following equation is written

$$\int\limits_0^2\left(\frac{\partial f}{\partial p}\right)_T dp+\frac{R}{\mu}\frac{k_T}{k_T-1}\bar{\eta}(T_2-T_0)+L_{\text{tech}}-L_{\text{fr}}^{\text{turb}}=0. \tag{26}$$

For the further manipulations, the prime need is to elucidate the result of integrating the partical derivative of the function f with respect to the pressure, at constant temperature.

2. To discover the integral $\int\limits_1^2\left(\frac{\partial f}{\partial p}\right)_T dp$, the notation $y = pv = f(p, T)$ is introduced and, hence, the following expression is obtained

$$dy=\left(\frac{\partial y}{\partial p}\right)_T dp+\left(\frac{\partial y}{\partial T}\right)_p dT.$$

Then

$$\int\limits_1^2\left[\frac{\partial(pv)}{\partial p}\right]_T dp=\int\limits_1^2\left(\frac{\partial y}{\partial p}\right)_T dp=\int\limits_1^2 dy-\int\limits_1^2\left(\frac{\partial y}{\partial T}\right)_p dT=y_2-y_1-\int\limits_1^2\left(\frac{\partial y}{\partial T}\right)_p dT$$

or

$$\int\limits_1^2\left[\frac{\partial(pv)}{\partial p}\right]_T dp=p_2v_2-p_1v_1-\int\limits_1^2\left[\frac{\partial(pv)}{\partial T}\right]_p dT. \tag{27}$$

From Eq. (12)

$$\omega=\frac{p}{R/\mu}\left(\frac{\partial v}{\partial T}\right)_p.$$

Writing $p(\partial v/\partial T)_p$ as $[\partial(pv)/\partial T]_p$, it follows that

$$\left[\frac{\partial(pv)}{\partial T}\right]_p=\frac{R}{\mu}\omega \tag{28}$$

or

$$\int_1^2 \left[\frac{\partial (pv)}{\partial T}\right] dT = \frac{R}{\mu}\bar{\omega}(T_2 - T_1). \tag{29}$$

Taking Eq. (29) into account, Eq. (27) is written in the form

$$\int_1^2 \left[\frac{\partial (pv)}{\partial p}\right]_T dp = p_2v_2 - p_1v_1 - \frac{R}{\mu}\bar{\omega}(T_2 - T_1). \tag{30}$$

Using the Lagrange formula, in which a relation is established between the increment of the function $f(T_2) - f(T_1)$ and the increment of the argument $(T_2 - T_1)$, Eq. (28) leads, after transformations, to the relation

$$(pv)_{T_2} - (pv)_{T_1} = \frac{R}{\mu}\bar{\omega}(T_2 - T_1) \tag{31}$$

or

$$(p_2v_2)_{T_2} - (p_2v_2)_{T_1} = \frac{R}{\mu}\bar{\omega}(T_2 - T_1). \tag{32}$$

Substituting Eq. (32) into Eq. (30) gives

$$\int_1^2 \left[\frac{\partial (pv)}{\partial p}\right] dp = p_2v_2 - p_1v_1 - [(p_2v_2)_{T_2} - (p_2v_2)_{T_1}]. \tag{33}$$

Appropriate transformations lead to the expression

$$\int_1^2 \left(\frac{\partial f}{\partial p}\right) dp = \frac{R}{\mu} T_1 [(Z_{2\mathrm{ef}})_{T_1} - (Z_{1\mathrm{ef}})_{T_1}], \tag{34}$$

where $(Z_{2\mathrm{ef}})_{T_1}$ is assumed at pressure p_2 but the same initial temperature T_1 [2].

3. To determine the drop in heat and the work done by the gas, the appropriate equations are obtained by substituting Eq. (34) successively into Eqs. (24), (25), and (26): the drop in heat corresponding to real expansion of the gas in the nozzle network

$$h_{\mathrm{N}} = \frac{w_1^2 - w_0^2}{2} = \frac{R}{\mu}\frac{k_T}{k_T - 1}\bar{\eta}T_0\left(1 - \frac{T_1}{T_0}\right) - \frac{R}{\mu}T_0[(Z_{1\mathrm{ef}})_{T_0} - (Z_{0\mathrm{ef}})_{T_0}] - L_{\mathrm{fr}}^{\mathrm{NN}}, \tag{35}$$

the work of 1 kg per gas at the working blades

$$L_{\mathrm{WB}} = \frac{R}{\mu}\frac{k_T}{k_T - 1}\bar{\eta}T_1\left(1 - \frac{T_2}{T_1}\right) - \frac{R}{\mu}T_1[(Z_{2\mathrm{ef}})_{T_1} - (Z_{0\mathrm{ef}})_{T_1}] - L_{\mathrm{fr}}^{\mathrm{WB}}, \tag{36}$$

the work actually developed by 1 kg of vapor inside the turbine

$$L_T = \frac{R}{\mu}\frac{k_T}{k_T - 1}\bar{\eta}T_0\left(1 - \frac{T_2}{T_0}\right) - \frac{R}{\mu}T_0[(Z_{2\mathrm{ef}})_{T_0} - (Z_{0\mathrm{ef}})_{T_0}] - L_{\mathrm{fr}}^{\mathrm{turb}}. \tag{37}$$

When $L_{\mathrm{fr}} = 0$, a process with a reversible adiabatic, i.e., on isoentropic process, will occur. Writing for the isoentropic curves an equation in terms of the temperature coefficient K_T in the form

$$\frac{T_{1t}}{T_0} = \left(\frac{p_1}{p_0}\right)^{(k_T - 1)/k_T}, \quad \frac{T_{2t}}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k_T - 1)/k_T}, \quad \frac{T_{2t}}{T_0} = \left(\frac{p_2}{p_0}\right)^{(k_T - 1)/k_T}, \tag{38}$$

the following relations are obtained: isoentropic or available drop in heat in the nozzle network

$$h_{\mathrm{AN}} = \frac{w_{1t}^2 - w_0^2}{2} = \frac{R}{\mu}\frac{k_T}{k_T - 1}\bar{\eta}T_0\left[1 - \left(\frac{p_1}{p_0}\right)^{(k_T - 1)/k_T}\right] - \frac{R}{\mu}T_0[(Z_{1\mathrm{ef}})_{T_0} - (Z_{0\mathrm{ef}})_{T_0}], \tag{39}$$

the work developed by the vapor flux at the working blades and equivalent to isoentropic expansion of the gas

$$(L_0)_{\mathrm{WB}} = \frac{R}{\mu}\frac{k_T}{k_T-1}\bar{\eta}T_1\left[1-\left(\frac{p_2}{p_1}\right)^{(k_T-1)/k_T} - \frac{R}{\mu}T_1\left[(Z_{2\mathrm{ef}})_{T_1}-(Z_{1\mathrm{ef}})_{T_1}\right]\right] - \frac{w_{2t}^2-w_1^2}{2}; \tag{40}$$

the isoentropic or available drop in heat in the turbine

$$H_0 = \frac{R}{\mu}\frac{k_T}{k_T-1}\bar{\eta}T_0\left[1-\left(\frac{p_2}{p_0}\right)^{(k_T-1)/k_T}\right] - \frac{R}{\mu}T_0\left[(Z_{2\mathrm{ef}})_{T_0}-(Z_{0\mathrm{ef}})_{T_0}\right]. \tag{41}$$

In deriving Eqs. (39)-(41), all the transformations, from the very beginning, have been performed with a view to obtaining dependences expressed in terms of p on T. This is done for the purpose of using the temperature index of the adiabatic k_T in subsequent work. In view of the slight change in $(k_T - 1)/k_T$, this leads to great expediency in the calculations, and ensures sufficient accuracy in the integrations.

Use of Eqs. (39)-(41) requires a knowledge of $(k_T - 1)/k_T$. As is evident from Eqs. (18) and (11), this entails, above all, finding the expression for the partial derivative $(\partial z_{\mathrm{ef}}/\partial T)_p$. The coefficient Z_{ef} is the product of two factors, i.e., $Z_{\mathrm{ef}} = Z\varepsilon$, where Z is the coefficient of compressibility, taking the nonideality of the gas into account, and the coefficient expresses the total number of moles as a result of the chemical reactions of the given dissociating gas.

NOTATION

p, v, T, ρ, volume, temperature, and density of the gas; R, universal gas constant; h, enthalpy; k_T, temperature coefficient of adiabatic or dissociating gas; k, adiabatic coefficient of perfect gas; μ, molecular mass; C_p, C_v, isobaric and isochoric mass specific heat; Z_{ef}, effective coefficient taking the influence both of dissociation and of nonideality of the gas into account.

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